SVD Radiosity

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In its simplest form, the "classic" radiosity method divides the surfaces of a diffusely reflective environment into n patches. Following radiative transfer theory, we have the radiosity equation:

$$\begin{bmatrix} M_{o1} \\ M_{o2} \\ \vdots \\ M_{on} \end{bmatrix} = \begin{bmatrix} 1-\rho_1 F_{11} & -\rho_1 F_{12} & \cdots & -\rho_1 F_{1n} \\ -\rho_2 F_{21} & 1-\rho_2 F_{22} & \cdots & -\rho_2 F_{2n} \\ \vdots & \vdots & \vdots & \vdots \\ -\rho_n F_{n1} & -\rho_n F_{n2} & \cdots & -\rho_n F_{nn} \end{bmatrix} \begin{bmatrix} M_1 \\ M_2 \\ \vdots \\ M_n \end{bmatrix} \tag{1}$$

where:

M_i is the exitance of patch *i*
M_{oi} is the emittance of patch *i*
ρ_i is the reflectance of patch *i*
F_{ij} is the form factor from patch *i* to patch *j*

Written in matrix form, the radiosity equation is:

$$\mathbf{M}_o = (\mathbf{I} - \mathbf{RF})\mathbf{M} \tag{2}$$

where **I** is the identity matrix, **R** is the diagonal reflectance matrix, and **F** is the form factor matrix. To solve for **M**, we can rewrite this equation as:

$$\mathbf{M} = (\mathbf{I} - \mathbf{RF})^{-1}\mathbf{M}_o \tag{3}$$

Now the matrix $\mathbf{Q} = (\mathbf{I} - \mathbf{RF})$ can be expressed in terms of its singular values $\mathbf{\Sigma}$ and left and right singular vectors **U** and **V** as:

$$\mathbf{Q} = \mathbf{U}\Sigma\mathbf{V}^T \tag{4}$$

and its inverse as:

$$\mathbf{Q}^{-1} = \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^T \tag{5}$$

Thus **M** can be solved directly as:

$$\mathbf{M} = (\mathbf{I} - \mathbf{RF})^{-1}\mathbf{M}_o == \mathbf{V}\mathbf{\Sigma}^{-1}\mathbf{U}^T\mathbf{M}_o \tag{6}$$

Given the singular value decomposition (SVD) expansion:

$$\mathbf{Q} = \sum_{i=1}^{n} \sigma_i u_i v_i^T \tag{7}$$

we also have:

$$\mathbf{M} = \sum_{i=1}^{k} \sigma_i^{-1} v_i u_i^T \mathbf{M}_o \tag{8}$$

where $k \le n$. If only a few of the singular values σ_i are small, then **M** may be approximated by only a few terms in $\mathbf{O}(kn)$ rather than $\mathbf{O}(n^2)$ time.